AD-A252 190 ‖‖‖‖‖‖‖‖‖ AGE

1. AGENCY US|		3. REPORT TYPE AND DATES COVERED
	May 14, 1992	Final 15 Aug 88 – 14 Oct 91

4. TITLE AND SUBTITLE	5. FUNDING NUMBERS
Geometric Theory of Infinite Dimensional Dynamical Systems. (U)	62301E 6611/00

6. AUTHOR(S)

Professor Dafermos, Mallet-Paret, Souganidis, Strauss

AFOSR-TR· 92 0545

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
Division of Applied Mathematics Brown University Providence, R.I. 02912	Final

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AFOSR Building 410 Bolling AFB DC 20332-6448	F49620-88-C-0129

11. SUPPLEMENTARY NOTES

DTIC
ELECTE
JUN 22 1992
S A D

12a. DISTRIBUTION/AVAILABILITY STATEMENT	12b. DISTRIBUTION CODE
Approved for Public Release; Distribution Unlimited	UL

13. ABSTRACT (Maximum 200 words)

A subject of investigation was the extent to which an entropy inequality (i.e., the Second Law of Thermodynamics) induces stabilization of solutions of hyperbolic systems of conservation laws. It was shown that the entropy inequality guarantees uniqueness of Lipschitz solutions within the class of BV solutions, provided that the entropy is convex just in certain directions compatible with the natural invariance of the system expressed in terms of "involutions". It was proven that BV solutions of strictly hyperbolic systems with shocks of moderate strength, which satisfy the Liu admissibility condition, minimize the rate of total entropy production. The theory of generalized characteristics for a single conservation law, developed earlier by the author, was applied to conservation laws with inhomogeneity and fading memory. The theory of generalized characteristics was developed for systems of conservations laws and was used to obtain information on the large time behavior of solutions. This theory was employed to establish uniqueness of solutions for special systems of conservation laws in which shock and rarefaction wave curves coincide.

14. SUBJECT TERMS			15. NUMBER OF PAGES
			7
			16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

WORK BY C. DAFERMOS

A subject of investigation was the extent to which an entropy inequality (i.e., the Second Law of Thermodynamics) induces stabilization of solutions of hyperbolic systems of conservation laws. In [1] it was shown that the entropy inequality guarantees uniqueness of Lipschitz solutions within the class of BV solutions, provided that the entropy is convex just in certain directions compatible with the natural invariance of the system expressed in terms of "involutions". In [7] it was proved that BV solutions of strictly hyperbolic systems with shocks of moderate strength, which satisfy the Liu admissibility condition, minimize the rate of total entropy production.
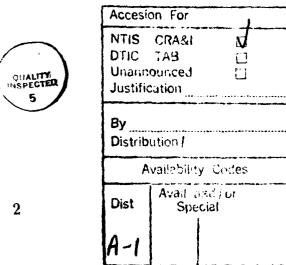
The theory of generalized characteristics for a single conservation law, developed earlier by the author, was applied in [3] and [5] to conservation laws with inhomogeneity and fading memory. In [8], the theory of generalized characteristics was developed for systems of conservation laws and was used to obtain information on the large time behavior of solutions. This theory was employed in [9] to establish uniqueness of solutions for special systems of conservation laws in which shock and rarefaction wave curves coincide.

References

[1] "Quasilinear Hyperbolic Systems with Involutions", *Arch. Rational Mech. Analysis*, 94 (1986), 373-389.

[2] "Estimates for Conservation Laws with Little Viscosity", *SIAM J. Math. Analysis*, 18 (1987), 409-421.

[3] "Trend to Steady State in a Conservation Law with Spatial Inhomogeneity", *Quart. Appl. Math.*, XLV (1987), 313-319.

[4] "Solutions in L^∞ for a Conservation Law with Memory", *Analyse Mathematique et Applications*, Gauthier-Villars, Paris (1988), 117-128.

[5] "Solutions with Shocks for Conservation Laws with Memory", *Amorphous Polymers and Non-Newtonian Fluids* (C. Dafermos, J. L. Ericksen, and D. Kinderlehrer, Eds.) Springer-Verlag, New York (1987), 33-55.

[6] "Hyperbolic conservation laws with memory", *Differential Equations* (C. Dafermos, G. Ladas and G. Papanicolaou, Eds.) Marcel Dekker, New York, (1989), 157-165.

[7] "Admissible wave fans in nonlinear hyperbolic systems", *Arch. Rational Mech. Analysis*, 106 (1989), 243-260.

[8] "Generalized characteristics in hyperbolic systems of conservation laws", *Arch. Rational Mech. Analysis*, 107 (1989), 127-155.

[9] "Generalized characteristics, uniqueness and regularity of solutions in a hyperbolic system of conservation laws", (with X. Geng), *Analyse non Linéaire*, 8 (1991), 231-269.

[10] "Generalized characteristics in hyperbolic systems of conservation laws with special coupling", (with X. Geng), *Proc. Royal Soc. Edinburgh*, 116A (1990), 245-278.

2

WORK BY J. MALLET-PARET

Research supported has involved several areas of differential equations, with special consideration given to simple and chaotic motions in infinite dimensional systems, delay differential equations, and singular perturbations, as outlined in the proposal. Several Ph.D. students have also been supported.

Our papers on higher dimensional Poincaré-Bendixson theorems [1,2] and generalizations [4], noted in a previous report, have been published.

Much of our recent efforts in delay differential equations have been directed toward understanding how monotonicity affects the qualitative behavior of solutions; such studies are a natural outgrowth of the earlier work [1,2]. Of particular interest have been equations with state-dependent delays. A prototypical problem is the singular perturbed equation

$$\varepsilon \dot{x}(t) = -x(t) + f(x(t-r)),$$

$$r = r(x(t)),$$

which arises in a number of models in biology, physiology, and economics. With Roger Nussbaum we have obtained new results [7] on the existence and certain qualitative properties (monotonicity and singular perturbation asymptotics) of slowly oscillating periodic solutions of state-dependent delay equations which confirm what is observed numerically. Our results indicate a definite *lack* of chaos in particular in the case of state-dependent delay.

With Donald Aronson and Martin Golubitsky [5], we have used global continuation methods for delay differential equations to prove existence of certain symmetric periodic solutions of a class of ODE's. The ODE's arise in systems of coupled Josephson junctions, and themselves admit a symmetry. Our novel approach establishes an equivalence between the ODE system and a related system of delay equations for solutions respecting an appropriate symmetry group.

3

Jiangping Fan, a student of Mallet-Paret's who obtain his Ph.D. in 1990 and who is now at the University of Rhode Island, has studied stability questions and developed a Floquet theory for singularly perturbed delay equations as above.

With Shui-Nee Chow and Kening Lu we have been studying the spectrum of various time-periodic parabolic PDE's, in the spirit of Floquet theory. Much of the motivation for this work comes from our earlier studies [1,2] of Poincaré-Bendixson theory for high dimensional systems. In a recently completed manuscript [6] the scalar problem

$$u_t = u_{xx} + a(t,x)u_x + b(t,x)u$$

is treated with either Dirichlet or Neumann boundary conditions on the interval $0 < x < \pi$. It is shown (in the Dirichlet case) the equivalence via a linear isomorphism to the time independent problem

$$u_t = u_{xx} + q(x)u$$

for some function $q(x)$. We are currently extending our study to the case of periodic boundary conditions (where new phenomena are expected owing to the complexity of the spectrum) and to the case of quasi-periodic coefficients.

Ying Huang, a student who obtained her Ph.D. under Mallet-Paret's direction in 1991 and who is now at the University of Minnesota, has studied the Floquet theory for retarded delay differential equations in a spirit analogous to the above.

References

[1] The Poincaré-Bendixson theorem for scalar reaction diffusion equations (with B. Fiedler), Arch. Rat. Mech. Anal., 107 (1989), 325-345.

[2] The Poincaré-Bendixson theorem for monotone cyclic feedback systems (with H.L. Smith), J. Dyn. Diff. Eq., 2 (1990), 367-421.

[3] A differential-delay equation arising in optics and physiology (with R.D. Nussbaum), SIAM Jour. Math. Anal. 20 (1989), 249-292.

[4] Stable periodic solutions for the hypercycle system (with J. Hofbauer and H. L. Smith), J. Dyn. Diff. Eq., 3 (1991), 423-436.

[5] Ponies on a merry-go-round in large arrays of Josephson junctions (with D.G. Aronson and M. Golubitsky), Nonlinearity, 4 (1991), 903-910.

[6] Floquet theory for parabolic differential equations: I, the time periodic case (with S. N. Chow and K. Lu), submitted for publication.

[7] Boundary layer phenomena for differential delay equations with state dependent time lags, I (with R.D. Nussbaum), to appear, Arch. Rat. Mech. Anal.

WORK BY W. A. STRAUSS

R. Glassey and I have almost completed our paper on the stability of
the relativistic maxwellian for the relativistic Boltzmann equation. We are
working on the coupled Vlasov-Maxwell-Boltzmann system.

M. Esteban and I have results on the stability of bound states in the
presence of a insulated sphere.

M. Beals and I have derived key L^p decay estimates for a wave equation
with a potential.

W. Craig, T. Kappeler and I have proven the gain of regularity of disper-
sive waves, both linear and nonlinear, in a very general context.

My current student, Y. Guo, has demonstrated the global existence of
weak solutions to the relativistic Vlasov-Maxwell system in the presence of
boundaries. The boundary conditions are either specular or partially absorb-
ing.

My current student, Y. Liu, has new results on the instability of certain
Boussinesq waves.

The following works have appeared since the last report:

- Nonlinear Wave Equations, NSF-CBMS Reg. Conf. Series No. 73,
  Amer. Math. Soc., Providence, 1989.

- Large velocities in the relativistic Vlasov-Maxwell equations, J. Fac.
  Sci. U. of Tokyo 1A, 36, No.3 (1989), 615-627 (with R. Glassey).

- Stability of solitary waves, Contemp. Math. 107 (1990), 123-129.

- Instability of a class of dispersive solitary waves, Proc. Royal Soc.
  Edinburgh 114A (1990), 195-212 (with P. Souganidis).

- Stability theory of solitary waves in the presence of symmetry II, J. Funct. Anal. 94 (1990), 308-348 (with M. Grillakis and J. Shatah).

- Infinite gain of regularity for dispersive evolution equations, in: Microlocal Analysis and Nonlinear Waves, IMA Volume 30, Springer-Verlag (1991), 47-50 (with W. Craig and T. Kappeler).

- On the derivatives of the collision map of relativistic particles, Transport Th. Statist. Phys. 20 (1991), 55-68 (with R. Glassey).

- Asymptotic stability of the relativistic maxwellian, preprint, (with R. Glassey).

- Infinite gain of regularity for dispersive wave equations, Proc. Conf. Nonlin. Varl. Probs. and PDEs, Elba, 1990 (with W. Craig and T. Kappeler).

- Time-decay estimates for a perturbed wave equation, Proc. Conf. St. Jean des Monts, 1991 (with M. Beals).

WORK BY P. SOUGANIDIS

Souganidis continued his work on the weak formulation of propagating fronts and/or its relations to applications like phase transitions, flame propagation etc. Among the completed projects are:

(i) Phase transitions and generalized motion by mean curvature (with L.C. Evans and H.M. Soner).

(ii) Front propagation and phase field theory (with G. Barles and H.M. Soner).

(iii) Uniqueness of rotationally symmetric surfaces moving by mean curvature (with H.M. Soner).

Souganidis and P.-L. Lions have established the convergence of second order accurate TVD schemes for scalar conservation laws and one dimensional Hamilton-Jacobi equations.